Orr et al. 2022 Superbubble Model

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1 Introduction

The evolution and blowout of superbubbles are key to understanding the mass, momentum, energy, and metals lost to galactic winds. However, these properties remain poorly constrained in observational studies (where they are challenging to infer from observables) and simulation-based studies (which require extremely high resolution to capture the various stages of superbubble evolution). An analytical framework for superbubble evolution and blowout can bridge this gap, offering valuable insights to better constrain outflow properties by coupling observational and theoretical approaches.

1.1 Model Description

1. The Orr et al. (2022) model implicitly accounts for prompt feedback like radiation pressure and stellar winds, which shape the IMF and set the local efficiency of star cluster formation as $N_{\rm SNe} \approx M_{\rm cl}/100M_\odot$ occur over a relatively short period of $t_{\rm SNe} \sim 40$ Myr (derived from Kroupa IMF with a cutoff mass of 8 $M_\odot$).

2. The geometry of this model ignores the vertical density gradient and assumes a slab geometry with the mean density $\bar{\rho}_g = \frac{\Sigma_g}{2H}$.

3. The Orr et al. (2022) model also assumes a momentum-driven superbubble because previous studies have shown that the mixing of cold material from subsumed clouds in a turbulent medium rapidly causes the hot interiors of bubbles to radiate away their thermal energy and enter the momentum-conserving phase, on the order of $\sim 10^4$ yr $\ll \rm t_{SNe}$.

1.2 Key Parameter Descriptions

1. $M_{\rm cl}$: stellar mass of the stellar cluster that forms instantaneously (star formation generally appears to stop in clouds before the first SN occurs at $t_d \sim 3$ Myr

2. t_d : delay timescale until first supernova

3. Star cluster formation efficiency $\epsilon_{\rm int} \approx \frac{\Sigma_g}{\Sigma_{\rm crit}}$, and $M_{\rm cl} = \epsilon_{\rm int} M_g \approx \pi H^2 \Sigma_g^2/\Sigma_{\rm crit}$. Here $\Sigma_{\rm crit} = 2800\ M_\odot \rm pc^{-2}$ (Grudić et al., 2018), which accounts for photoionizing radiation.

4. The disk scale height $H \approx \sigma/\Omega = \sigma R/v_c$. The typical value is 10 km s^{-1} $\times$ 5 kpc / 200 km s^{-1}.

5. The model assumes that the Toomre parameter $\widetilde{Q}_{\rm gas} \approx \frac{H}{GM/\sigma^2} \approx \frac{\sigma^2 H}{G\Sigma H^2} \approx \frac{\sigma\Omega}{G\Sigma} = 1$ $\left(\text{the exact form is } \widetilde{Q}_{\rm gas} = \frac{\sqrt{2}\sigma\Omega}{\pi G \Sigma_{\rm disk}}\right)$. Since we know that $\Sigma_{\rm disk} = \Sigma_g + \Sigma_*$, $\tilde{f}_g = \frac{\Sigma_g}{\Sigma_g + \Sigma_*}$, and $\Sigma_{\rm disk} = \Sigma_g/\tilde{f}_g$, $\Sigma_g = \frac{\sqrt{2}\sigma\Omega\tilde{f}_g}{\pi G}$ when $\widetilde{Q}_{\rm gas} = 1$.

1.3 Derivation

1.3.1 Momentum Injection of SNe $P_{\rm SNe}$

Momentum of the bubble: $P_b = \frac{4}{3}\pi R_b^3 \bar{\rho}_g \frac{dR_b}{dt}$, where $V_b = dR_b/dt$. This bubble momentum is balanced by the momentum injection from the SNe of the central star cluster.

Since SNe are distributed in time, the time-delay distribution of core-collapse SNe can be derived by convolving the high-end mass slope of the IMF ($dN_*/dM_* \propto M_*^{-2.35}$) with an estimate for high-mass stellar lifetimes

($t_* \propto M_*/L_*, L_* \propto M_*^3$). The derivative of the number of SNe to time can be derived as follows:

$$\begin{aligned} t_* &\propto \frac{M_*}{M_*^{3.5}} \propto M_*^{-2.5} \Rightarrow \frac{dM_*}{dt} \propto t^{-7/5} \left(M_* \propto t^{-2/5}\right) \\ \Rightarrow \frac{dN_{\rm SN}}{dt} &= \frac{dN}{dM_*}\frac{dM_*}{dt} = M_*^{-2.35}\ t^{-7/5} \propto (t^{-2/5})^{-2.35}\ t^{-7/5} \propto t^{-0.46}. \end{aligned} \tag{1}$$

The momentum injection from the SNe of the central star cluster as a function of time is

$$P_{\rm SNe}(t) = \int_0^t \frac{dN_{\rm SN}}{dt'}\ \left(\frac{P}{m_*}\right) dt', \tag{2}$$

where $\left(\frac{P}{m_*}\right)$ is the fiducial momentum injected by a single SN.
Because $N_{\rm SNe} = M_{\rm cl}/100M_\odot$ over $t_{\rm SNe}$ and $dN_{\rm SN}/dt = Ct^{-\alpha}$ (C is a constant),

$$\int_0^{t_{\rm SNe}} \frac{dN_{\rm SN}}{dt'}dt' = \frac{M_{\rm cl}}{100M_\odot} \Rightarrow \frac{C}{-\alpha+1}t^{-\alpha+1}\Big|_0^{t_{\rm SNe}} = \frac{C}{1-\alpha}t_{\rm SNe}^{1-\alpha},\ C = \frac{M_{\rm cl}}{100M_\odot}\frac{1-\alpha}{t_{\rm SNe}^{1-\alpha}}. \tag{3}$$

If we absorb the factor of $(1/100M_\odot)$ into (P/m_*) as $(P/m_*)_0$ (i.e., the fiducial momentum injection normalized per 100 $M_\odot$), we can derive the expression of $P_{\rm SNe}$ in the following way

$$\begin{aligned} P_{\rm SNe}(t) &= M_{\rm cl}\left(\frac{P}{m_*}\right)_0 \frac{(1-\alpha)}{t_{\rm SNe}^{1-\alpha}}\int_0^t t'^{-\alpha}dt' \\ P_{\rm SNe}(t) &= M_{\rm cl}\left(\frac{P}{m_*}\right)_0 \begin{cases} (t/t_{\rm SNe})^{1-\alpha}, & 0 < t < t_{\rm SNe} \\ 1, & t > t_{\rm SNe} \end{cases} = \frac{2}{3}\pi R_b^3 \frac{\Sigma_g}{H}\frac{dR_b}{dt}. \end{aligned} \tag{4}$$

Here we ignore the short delay-time term t_d in the integral $\int_0^t (t'+t_d)^{-\alpha}\,dt'$.

1.3.2 Superbubble Radius R_b and Velocity v_b

Since we know that $\Sigma_g^2 = M_{\rm cl}\Sigma_{\rm crit}/\pi H^2$

$$R_b^3\frac{dR_b}{dt} = \frac{3}{2} \underbrace{\frac{M_{\rm cl}H}{\pi\Sigma_g}}_{\frac{\Sigma_g^2}{\Sigma_g}H\frac{H^2}{\Sigma_{\rm crit}} = \Sigma_g\frac{H^3}{\Sigma_{\rm crit}}} \left(\frac{P}{m_*}\right)_0 \begin{cases} \left(\frac{t}{t_{\rm SNe}}\right)^{1-\alpha}, & 0 < t < t_{\rm SNe} \\ 1, & t > t_{\rm SNe} \end{cases} \tag{5}$$

We can then integrate both sides of Equation 5 with respect to t,

$$\begin{aligned} \int_0^t R_b^3 dR_b &= \int_0^t \left[\frac{3}{2}\Sigma_g\frac{H^3}{\Sigma_{\rm crit}}\left(\frac{P}{m_*}\right)_0 \begin{cases} \left(\frac{t}{t_{\rm SNe}}\right)^{1-\alpha}, & 0 < t < t_{\rm SNe} \\ 1, & t > t_{\rm SNe} \end{cases}\right] dt \\ \frac{R_b(t)^4}{4} &= \frac{3}{2}\Sigma_g\frac{H^3}{\Sigma_{\rm crit}}\left(\frac{P}{m_*}\right)_0 \begin{cases} t_{\rm SNe}^{-(1-\alpha)}\frac{t^{2-\alpha}}{2-\alpha}, & 0 < t < t_{\rm SNe} \\ t_{\rm SNe}^{(1-\alpha)}\frac{t_{\rm SNe}^{2-\alpha}}{2-\alpha} + t - t_{\rm SNe}, & t > t_{\rm SNe} \end{cases}, \end{aligned} \tag{6}$$

which can be reduced to the format following Orr et al. (2022).

$$R_b(t) = H\left[6\frac{\Sigma_g}{\Sigma_{\rm crit}}\frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{1/4} \begin{cases} \frac{1}{(2-\alpha)^{1/4}}\left(\frac{t}{t_{\rm SNe}}\right)^{\frac{2-\alpha}{4}}, & 0 < t < t_{\rm SNe} \\ \left[\frac{t}{t_{\rm SNe}} - \left(\frac{1-\alpha}{2-\alpha}\right)\right]^{1/4}, & t > t_{\rm SNe} \end{cases} \tag{7}$$

The superbubble velocity can be derived by taking the derivative of Equation 7 with respect to t:

$$v_b = \frac{dR_b}{dt} = H\left[6\frac{\Sigma_g}{\Sigma_{\rm crit}}\frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{1/4} \begin{cases} \frac{1}{(2-\alpha)^{1/4}}\frac{2-\alpha}{4}\left(\frac{t}{t_{\rm SNe}}\right)^{\frac{-(2+\alpha)}{4}}\frac{1}{t_{\rm SNe}}, & 0 < t < t_{\rm SNe} \\ \frac{1}{4}\left[\frac{t}{t_{\rm SNe}} - \left(\frac{1-\alpha}{2-\alpha}\right)\right]^{-3/4}\frac{1}{t_{\rm SNe}}, & t > t_{\rm SNe} \end{cases} \tag{8}$$

Equation 8 can be also reorganized into the format following Orr et al. (2022).

$$v_b(t) = \frac{H/t_{\rm SNe}}{4}\left[6\frac{\Sigma_g}{\Sigma_{\rm crit}}\frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{1/4} \begin{cases} (2-\alpha)^{3/4}\left(\frac{t}{t_{\rm SNe}}\right)^{-(2+\alpha)/4}, & 0 < t < t_{\rm SNe} \\ \left[\frac{t}{t_{\rm SNe}} - \left(\frac{1-\alpha}{2-\alpha}\right)\right]^{-3/4}, & t > t_{\rm SNe} \end{cases} \tag{9}$$

1.3.3 Four Scenarios of Superbubble Evolutions

1. PBO case: "Powered Breakout"

$$R_b = H \text{ when } t_{\rm BO} < t_{\rm SNe}$$

2. CBO case: "Coasting (unpowered) Breakout"

$$R_b = H \text{ when } t_{\rm BO} > t_{\rm SNe}$$

3. CF case: "coasting (unpowered) Fragmentation"

$$v_b < \sigma \text{ when } R_b\,(v_b = \sigma) < H \;\&\; t\,(v_b = \sigma) > t_{\rm SNe}$$

4. PS case: "Powered Stall"

$$v_b < \sigma \text{ when } R_b\,(v_b = \sigma) < H \;\&\; t\,(v_b = \sigma) < t_{\rm SNe}$$

Border of PBO/PS case:

When $t < t_{\rm SNe}$ & $R_b = H$,

$$R_b = H\left[6\frac{\Sigma_g}{\Sigma_{\rm crit}}\frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{1/4}\frac{1}{(2-\alpha)^{1/4}}\left(\frac{t}{t_{\rm SNe}}\right)^{\frac{2-\alpha}{4}} = H. \tag{10}$$

And based on the boundary condition $v_b = \sigma$ when $t < t_{\rm SNe}$, we have

$$v_b = \frac{H/t_{\rm SNe}}{4}\left[6\frac{\Sigma_g}{\Sigma_{\rm crit}}\frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{1/4}(2-\alpha)^{3/4}\left(\frac{t}{t_{\rm SNe}}\right)^{-(2+\alpha)/4} = \sigma \tag{11}$$

We can then reorganize Equations 10 and 11 to derive the expression of t in terms of other key parameters. From Equations 10 and 11, we have

$$t = t_{\rm SNe}(2-\alpha)^{1/(2-\alpha)}\left[6\frac{\Sigma_g}{\Sigma_{\rm crit}}\frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{\frac{-1}{2-\alpha}} \tag{12}$$

and

$$t = t_{\rm SNe}\left(\frac{H/t_{\rm SNe}}{4\sigma}\right)^{4/(2+\alpha)}\left[6\frac{\Sigma_g}{\Sigma_{\rm crit}}\frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{1/(2+\alpha)}(2-\alpha)^{\frac{3}{2+\alpha}}, \tag{13}$$

respectively.

Equating Equations 12 and 13, we can obtain

$$\left[6\frac{\Sigma_g}{\Sigma_{\rm crit}}\frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{\left(\frac{1}{2+\alpha}+\frac{1}{2-\alpha}\right)} = (2-\alpha)^{\left(\frac{1}{(2-\alpha}-\frac{3}{2+\alpha}\right)}\left(\frac{4\sigma}{H/t_{\rm SNe}}\right)^{\frac{4}{2+\alpha}} \tag{14}$$

Since we know that $\Sigma_g = \frac{\sqrt{2}\sigma\Omega\tilde{f}_g}{\pi G}$,

$$\left[\frac{6\sqrt{2}\sigma\Omega\widetilde{f}_g}{\pi G\Sigma_{\rm crit}}\frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{\left(\frac{1}{2+\alpha}+\frac{1}{2-\alpha}\right)} = (2-\alpha)^{\left(\frac{1}{2-\alpha}-\frac{3}{2+\alpha}\right)}\left(\frac{4\sigma}{H/t_{\rm SNe}}\right)^{\frac{4}{2+\alpha}} \tag{15}$$

$$\tilde{f}_g^{\frac{4}{4-\alpha^2}} = (2-\alpha)^{\frac{4(\alpha-1)}{4-\alpha^2}}\left[\frac{6\sqrt{2}\sigma\Omega}{\pi G\Sigma_{\rm crit}}\frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{-\frac{4}{4-\alpha^2}}\left(\frac{4\sigma}{H/t_{\rm SNe}}\right)^{\frac{4}{2+\alpha}} \tag{16}$$

$$\tilde{f}_g = (2-\alpha)^{(\alpha-1)}\left[\frac{\pi G\Sigma_{\rm crit}}{6\sqrt{2}\sigma\Omega}\frac{H/t_{\rm SNe}}{(P/m_*)_0}\right]\left(\frac{4\sigma}{\underbrace{H/t_{\rm SNe}}_{4\Omega t_{\rm SNe}}}\right)^{\frac{4}{2+\alpha}\frac{4-\alpha^2}{4}} \tag{17}$$

$$\tilde{f}_g = 4\Omega t_{\rm SNe}\left(\frac{4\Omega t_{\rm SNe}}{2-\alpha}\right)^{1-\alpha}\left[\frac{\pi G\Sigma_{\rm crit}}{6\sqrt{2}}\frac{H}{\sigma\Omega}\frac{1}{t_{\rm SNe}}\frac{1}{(P/m_*)_0}\right] \tag{18}$$

Because $\frac{H}{\sigma} = \frac{1}{\Omega}$,

$$\tilde{f}_g = \frac{\sqrt{2}\pi G}{3} \frac{\Sigma_{\rm crit}}{(P/m_*)_0} \left(\frac{4\Omega t_{\rm SNe}}{2-\alpha}\right)^{1-\alpha} \frac{1}{\Omega} \tag{19}$$

Border of PBO/CBO case:

Based on the boundary conditions of $R_b = H$ & $t = t_{\rm SNe}$, we have

$$t = t_{\rm SNe}(2-\alpha)^{1/(2-\alpha)} \left[6\frac{\Sigma_g}{\Sigma_{\rm crit}} \frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{\frac{-1}{2-\alpha}} \quad \& \; t = t_{\rm SNe} \tag{20}$$

$$t_{\rm SNe} = t_{\rm SNe}(2-\alpha)^{1/(2-\alpha)} \left[6\frac{\Sigma_g}{\Sigma_{\rm crit}} \frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{\frac{-1}{2-\alpha}} \tag{21}$$

$$1 = (2-\alpha)^{\frac{1}{2-\alpha}} \left[6\frac{\Sigma_g}{\Sigma_{\rm crit}} \frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{\frac{-1}{2-\alpha}} \tag{22}$$

$$\left[6\frac{\Sigma_g}{\Sigma_{\rm crit}} \frac{(P/m_*)_0}{H/t_{\rm SNe}}\right] = (2-\alpha); \; 6\frac{\sqrt{2}\sigma\Omega\tilde{f}_g}{\pi G \Sigma_{\rm crit}} \frac{(P/m_*)_0}{H/t_{\rm SNe}} = 2-\alpha \tag{23}$$

$$\tilde{f}_g = \frac{\sqrt{2}\pi G}{3} \frac{\Sigma_{\rm crit}}{(P/m_*)_0} \left(\frac{2-\alpha}{4\Omega t_{\rm SNe}}\right) \frac{1}{\Omega} \tag{24}$$

Border of CF/PS case:

Based on the boundary conditions $v_b \leq \sigma$ when $t = t_{\rm SNe}$, we can rewrite Equation 13 as

$$t_{\rm SNe} \geq t_{\rm SNe} \left(\frac{H/t_{\rm SNe}}{4\sigma}\right)^{4/(2+\alpha)} \left[6\frac{\Sigma_g}{\Sigma_{\rm crit}} \frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{1/(2+\alpha)} (2-\alpha)^{\frac{3}{2+\alpha}} \tag{25}$$

$$1 \geq \left(\frac{H/t_{\rm SNe}}{4\sigma}\right)^{4(2+\alpha)} \left[6\frac{\Sigma_g}{\Sigma_{\rm crit}} \frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{1/(2+\alpha)} (2-\alpha)^{\frac{3}{2+\alpha}} \tag{26}$$

$$1 \geq \left(\frac{1}{4\Omega t_{\rm SNe}}\right)^4 \left(6\frac{\Sigma_g}{\Sigma_{\rm crit}} \frac{(P/m_*)_0}{H/t_{\rm SNe}}\right) (2-\alpha)^3 \tag{27}$$

$$1 \geq \left(\frac{1}{4\Omega t_{\rm SNe}}\right)^4 \left(6\frac{\sqrt{2}\sigma\Omega\tilde{f}_g}{\pi G \Sigma_{\rm crit}} \frac{(P/m_*)_0}{H/t_{\rm SNe}}\right) (2-\alpha)^3 \tag{28}$$

$$\tilde{f}_g \leq (4\Omega t_{\rm SNe})^4 (2-\alpha)^{-3} \frac{\Sigma_{\rm crit}}{(P/m_*)_0} \frac{\pi G}{6\sqrt{2}} \frac{H}{\sigma} (\Omega t_{\rm SNe})^{-1} \tag{29}$$

$$\tilde{f}_g \leq \frac{4}{6\sqrt{2}} \pi G \frac{\Sigma_{\rm crit}}{(P/m_*)_0} \left(\frac{4\Omega t_{\rm SNe}}{2-\alpha}\right)^3 \frac{1}{\Omega} \tag{30}$$

$$\tilde{f}_g \leq \frac{\sqrt{2}}{3} \pi G \frac{\Sigma_{\rm crit}}{(P/m_*)_0} \left(\frac{4\Omega t_{\rm SNe}}{2-\alpha}\right)^3 \frac{1}{\Omega} \tag{31}$$

Border of CBO/CF case:

Based on the boundary conditions $R_b = H$ and $v_b = \sigma$ when $t > t_{\rm SNe}$, we have

$$H \left[6\frac{\Sigma_g}{\Sigma_{\rm crit}} \frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{1/4} \left[\frac{t}{t_{\rm SNe}} - \left(\frac{1-\alpha}{2-\alpha}\right)\right]^{1/4} = H \tag{32}$$

$$\frac{t}{t_{\rm SNe}} = \left(6\frac{\Sigma_g}{\Sigma_{\rm crit}} \frac{(P/m_*)_0}{H/t_{\rm SNe}}\right)^{-1} + \left(\frac{1-\alpha}{2-\alpha}\right) \tag{33}$$

$$\tag{34}$$

and

$$\frac{H/t_{\rm SNe}}{4} \left[6\frac{\Sigma_g}{\Sigma_{\rm crit}} \frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{1/4} \left[\frac{t}{t_{\rm SNe}} - \left(\frac{1-\alpha}{2-\alpha}\right)\right]^{-3/4} = \sigma \tag{35}$$

$$\left(\frac{H/t_{\rm SNe}}{4\sigma}\right)^4 \left[6\frac{\Sigma_g}{\Sigma_{\rm crit}} \frac{(P/m_*)_0}{H/t_{\rm SNe}}\right] = \left[\frac{t}{t_{\rm SNe}} - \left(\frac{1-\alpha}{2-\alpha}\right)\right]^3 \tag{36}$$

$$\frac{t}{t_{\rm SNe}} = \left(\frac{H/t_{\rm SNe}}{4\sigma}\right)^{4/3} \left[6\frac{\Sigma_g}{\Sigma_{\rm crit}} \frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{1/3} + \left(\frac{1-\alpha}{2-\alpha}\right) \tag{37}$$

Combining Equations 33 and 37, we can obtain

$$\left(6\frac{\Sigma_g}{\Sigma_{\rm crit}}\frac{(P/m_*)_0}{H/t_{\rm SNe}}\right)^{-1}+\left(\frac{1-\alpha}{2-\alpha}\right)=\left(\frac{H/t_{\rm SNe}}{4\sigma}\right)^{4/3}\left[6\frac{\Sigma_g}{\Sigma_{\rm crit}}\frac{(P/m_*)_0}{H/t_{\rm SNe}}\right]^{1/3}+\left(\frac{1-\alpha}{2-\alpha}\right) \tag{38}$$

$$\left(6\frac{\Sigma_g}{\Sigma_{\rm crit}}\frac{(P/m_*)_0}{H/t_{\rm SNe}}\right)^{-1}=\left(\frac{H/t_{\rm SNe}}{4\sigma}\right)=\frac{1}{4\Omega t_{\rm SNe}} \tag{39}$$

$$\left(6\frac{\sqrt{2}\sigma\Omega\tilde{f}_g}{\pi G\Sigma_{\rm crit}}\frac{(P/m_*)_0}{H/t_{\rm SNe}}\right)^{-1}=\frac{1}{4\Omega t_{\rm SNe}} \tag{40}$$

$$\tilde{f}_g=\frac{4\pi G}{6\sqrt{2}}\frac{\Sigma_{\rm crit}}{(P/m_*)_0}\frac{1}{\Omega}=\frac{\sqrt{2}\pi G}{3}\frac{\Sigma_{\rm crit}}{(P/m_*)_0}\frac{1}{\Omega} \tag{41}$$

Critical Ω

The critical Ω, $\Omega_{\rm crit}$, can be determined from the point of intersection for the four scenarios in the $\tilde{f}_g-\Omega$ space, which can be easily derived as $\Omega_{\rm crit}=\frac{2-\alpha}{4t_{\rm SNe}}$.

Optional:

If we want to plot the $\dot{\Sigma}_*-\Omega$ space instead,

$$\dot{\Sigma}_*\approx\frac{M_{\rm cl}\Omega}{\pi H^2}\approx\frac{\left(\pi H^2\Sigma_g^2/\Sigma_{\rm crit}\right)\Omega}{\pi H^2}=\frac{2\sigma^2\Omega^3\tilde{f}_g^2}{\pi^2G^2\Sigma_{\rm crit}}, \tag{42}$$

where $M_{\rm cl}\cong\frac{\pi H^2\Sigma_g^2}{\Sigma_{\rm crit}}$ and $\Sigma_g=\frac{\sqrt{2}\sigma\Omega\tilde{f}_g}{\pi G}$. The Orr et al. (2022) case boundaries are shown in Figure 1 for both the $\tilde{f}_{\rm gas}-\Omega$ space (left panel) and the $\dot{\Sigma}_*-\Omega$ space (right panel).

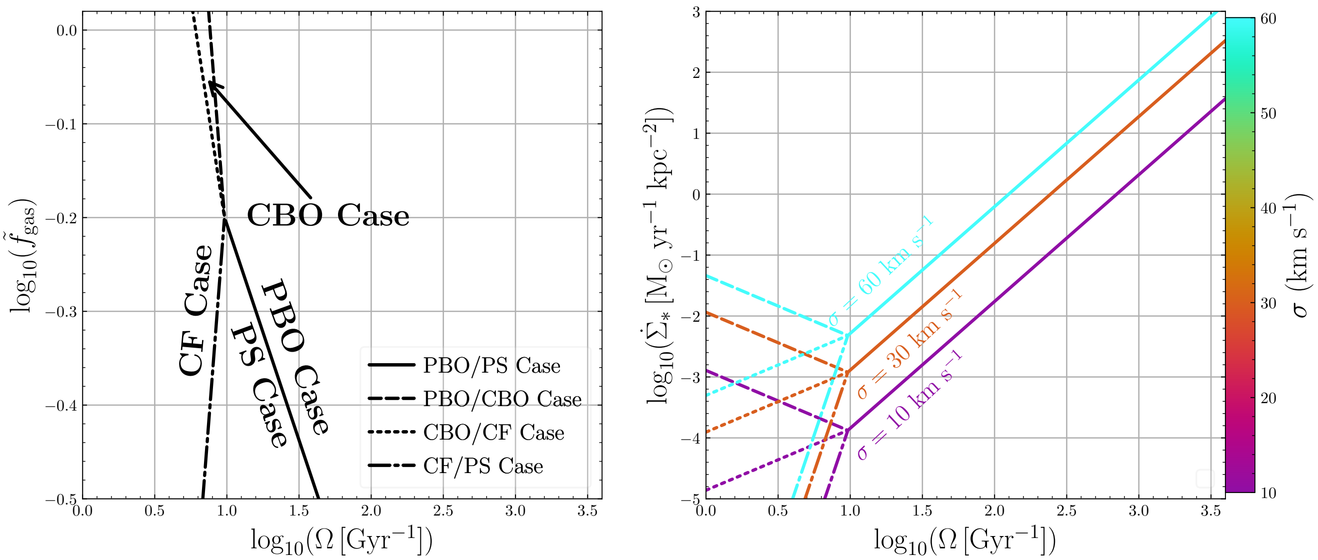

Figure 1: Orr et al. (2022) case boundaries with $\tilde{f}_{\rm gas}$ vs. Ω plotted on the *left* panel and $\dot{\Sigma}_*$ vs. Ω on the *right* panel (as a function of σ) .

References

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