Analytical Chemical Evolution Model

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1 Introduction

Chemical evolution is a complex and multifaceted process that plays a crucial role in understanding the composition and dynamics of galaxies. With a multitude of factors influencing this process, from nucleosynthesis to the accretion and ejection of material in galactic bodies, developing a comprehensive and accurate model can be a significant challenge.

In this document, we delve into the mathematics of the analytical chemical evolution model proposed by Edmunds and Pagel (1984) and Erb (2008). This approach allows us to derive relationships between various elements and understand their abundance over time.

We will explore concepts such as the total system mass, gas mass, and the mass locked in stars. Our approach involves the instantaneous recycling approximation and explores how different factors, such as the rate of accretion, ejection, and star formation, affect chemical evolution. Through rigorous mathematical derivation, we aim to provide insights into how the abundance of specific elements relates to the gas fraction and vice versa.

The subsequent sections outline the derivation of key equations that constitute the foundation of the analytical chemical evolution model.

2 Derivation

We primarily follow the notation from *Nucleosynthesis And Chemical Evolution of Galaxies* (Pagel, 2009). We want to find out the relation between the abundance Z of the element of interest and the gas fraction μ. We define the total system mass as M, the mass of gas as g, and the mass existing in the form of stars as s (i.e., $s = \alpha S$, where α is the fraction of mass remaining locked in stars and S is the total stellar mass). Based on this definition, we have

$$M = g + s \tag{1}$$

and

$$\frac{dM}{dt} = F - E \tag{2}$$

where F is the accretion rate of material from outside the system, and E is the rate of ejection, e.g., in a galactic wind.

If we want to find out the gas mass changes over time, we could write $\frac{dg}{dt}$ in terms of F and E:

$$\frac{dg}{dt} = F - E - \frac{ds}{dt} \tag{3}$$

, so

$$\frac{dg}{ds} = \frac{(F-E)\ dt}{ds} - 1 = \frac{F-E}{\alpha\psi} - 1 \tag{4}$$

where ψ is the star formation rate and $S(t) = \int_0^t \psi(t')dt'$.

Adopting the instantaneous recycling approximation (assume that all processes involving stellar evolution, nucleosynthesis, and recycling take place instantaneously on the timescale of galactic evolution), which is a good assumption for the elements like oxygen, and a homogeneous ISM, the gas abundance of a stable robust element is determined by the following formula:

$$\frac{d(gZ)}{dS} = (p\alpha) - (Z\alpha) - Z_E\frac{E}{\psi} + Z_F\frac{F}{\psi} \tag{5}$$

The RHS terms can be interpreted as new production (p is the true stellar yield), lock-up in stars, loss in galactic wind, and gain from inflows or accretions, respectively. For a homogeneous wind with $Z_E = Z$, we can divide the equation above by α and get the following equation:

$$\frac{d(gZ)}{ds} = p - Z - Z\frac{E}{\alpha\psi} + Z_F\frac{F}{\alpha\psi} = p - Z(1+\frac{E}{\alpha\psi}) + Z_F\frac{F}{\alpha\psi} \tag{6}$$

Since we know that

$$\frac{d(gZ)}{ds} = Z\frac{dg}{ds} + g\frac{dZ}{ds} = Z\left(\frac{F-E}{\alpha\psi} - 1\right) + g\frac{dZ}{ds} \tag{7}$$

so

$$g\frac{dZ}{ds} = p - Z(1+\frac{E}{\alpha\psi}) + Z_F\frac{F}{\alpha\psi} - Z\left(\frac{F-E}{\alpha\psi} - 1\right) = p + (Z_F - Z)\frac{F}{\alpha\psi} = p + (Z_F - Z)\frac{f_i}{\alpha}, \tag{8}$$

where we assume the ratio between the inflow rate and the SFR is $f_i = F/\psi$.

Solving Equation 8 above (assume the inflowing gas has negligible metallicity $Z_F \sim 0$ and the initial condition $Z(s=0) = 0$, we have

$$Z(s) = \frac{p\alpha}{f_i}\left[1 - \left(\frac{g}{M_i}\right)^{f_i/(\alpha - f_i + f_o)}\right] \tag{9}$$

Following Erb (2008) or Edmunds and Pagel (1984), if we want to write Z in terms of the gas fraction μ, we first need to know how the gas fraction relates to the ratio of the current to the initial gas mass (i.e., g/M_i). We can first write down the equation of the gas mass:

$$g = M_i + S(-\alpha + f_i - f_o) \tag{10}$$

so

$$g + s = M_i + \left(\frac{f_i - f_o}{\alpha}\right)s \tag{11}$$

Since the gas fraction $\mu = \frac{g}{g+s}$, so we have

$$g + s = \frac{g}{\mu} = M_i + \left(\frac{f_i - f_o}{\alpha}\right)s \tag{12}$$

so

$$\frac{g}{M_i} = \mu\left(1 + \frac{1}{M_i}\left(\frac{f_i - f_o}{\alpha}\right)s\right) = \mu\left(1 + \frac{s(f_i - f_o)/\alpha}{g + (1+\frac{f_o - f_i}{\alpha})s}\right) = \mu\left(\frac{g+s}{g + (1+\frac{f_o - f_i}{\alpha})s}\right) \tag{13}$$

Because we know that $g + s = g/\mu$, we can re-write Equation 13 as follows:

$$\frac{g}{M_i} = \mu\left(\frac{g/\mu}{g + (1+\frac{f_o - f_i}{\alpha})s}\right) \tag{14}$$

Dividing the RHS of Equation 14 by g/μ and simplifying the terms, we can get

$$\frac{g}{M_i} = \frac{\mu}{1 + (1-\mu)\frac{f_o - f_i}{\alpha}}, \tag{15}$$

which is the same as Equation (13) in Erb (2008). We can then plug Equation 15 into Equation 9 to determine how the abundance of this specific element Z relates to the gas fraction μ. This yields:

$$Z(\mu) = \frac{p\alpha}{f_i}\left[1 - \left(\frac{\mu}{1 + (1-\mu)\frac{f_o - f_i}{\alpha}}\right)^{f_i/(\alpha - f_i + f_o)}\right]. \tag{16}$$

Equation 16 describes the abundance of a specific element as a function of the gas fraction μ. This allows us to investigate how the elemental abundance evolves as the gas fraction of the system changes.

References

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